Proof by contradiction- Its ~~the~~ a method of proving where we show that a statement is false by contradicting it and finding an ~~statem~~ example to which the statement does not adide by.

Proofs-

1 Why is $\sqrt{2}$ irrational?

2 How is $\sqrt{4}$ rational?

3 How is $\sqrt{63}$ irrational?

4 How is $\sqrt{m}$ irrational when m is not a perfect square?

1 - Why is $\sqrt{2}$ irrational?

If $\sqrt{2}$ is rational, then-

$\sqrt{2}$ can be ~~re presen~~ represented a $\frac{p}{q}$, with p & q being co-primes. $\therefore \sqrt{2} = \frac{p}{q}$

How can we make $\sqrt{2}$ as 2? By squaring both sides

$= (\sqrt{2})^2 = \frac{p^2}{q^2} \Rightarrow 2 = \frac{p^2}{q^2}$

We can re-write $2 = \frac{p^2}{q^2}$ as $\rightarrow p^2 = 2(q^2)$

Any whole number multiplie'd by 2 = even number

$\therefore p^2$ is an even number.

What is the difference between $(odd)^2$ & $(even)^2$? $(odd)^2 = odd$

$(even)^2 = even$

How can a rational number be represented $\frac{p}{q}$, with p & q as co-primes?

What are co-primes?

2 numbers whose common factor is only 1

: because p^2 is even, we can say that $\boxed{p \text{ is also even.}}$

Step 4- We can again re-arrange $2 = \frac{p^2}{q^2}$ to form $\rightarrow \frac{p^2}{2} = q^2$

$= q^2$ is even. (any number divided by 2 which results in a whole number = the whole number is even)

$(\text{even})^2 = \text{even}$

$= \boxed{q = \text{even}}$

Step 5- Because p & q are even, they can be further simplified & so, are not co-primes.

$= \frac{p}{q} \neq$ co-prime $= \sqrt{2} \neq$ rational

2- How is $\sqrt{4}$ rational?

Step 1- How can $\sqrt{4}$ be represented? As $\frac{p}{q}$

$\therefore \sqrt{4} = \frac{p}{q}$ → How can we get four & not $\sqrt{4}$? We can square on both sides

$\therefore (\sqrt{4})^2 = \frac{p^2}{q^2}$ $\therefore$ $4 = \frac{p^2}{q^2}$

Step 2- We can say that 4 is equal to a^2 (because it is perfect square)

$\therefore a^2 = 4 = \frac{p^2}{q^2} = a^2 = \frac{p^2}{q^2}$ $\therefore a = \frac{p}{q}$

Step 3. a is a whole number written a $\frac{p}{q}$ & thus, we can say that $\sqrt{4}$ is rational.

3- How is $\sqrt{63}$ irrational?

Step 1- How can we represent $\sqrt{63}$? As $\frac{p}{q}$

How can we make $\sqrt{63}$ as 63? By squaring of both sides.

$= \sqrt{63} = \frac{p}{q}$ $= (\sqrt{63})^2 = \frac{p^2}{q^2} = 63 = \frac{p^2}{q^2}$

Step 2- By dividing what type of numbers do we get 63? $\frac{odd}{odd} = odd$ $\frac{odd}{even} = \text{not whole}$

Only if we divide $\frac{odd}{odd}$ do we get 63 (an odd)

$\frac{even}{even} = even$ $\frac{even}{odd} = even$

So, we can say that p^2 & q^2 are odd.

There are no exceptions. Although examples like $\frac{21}{5}$ are co-primes, they don't result in a whole number.

= only if ~~the~~ the numbers are not co-primes, will we get a whole no. (63)

4 - How is $\sqrt{m}$ irrational when m is not a perfect square?

What will the square root of a non-perfect square be? decimal number

What (decimal no.)^2 results in whole number (with 0)? none →

∴ No whole number OR decimal number when multiplied by itself gives a whole number.

This is because m lies between 2 perfect squares.

If this occurs, ~~then~~ then $\sqrt{m}$ is always irrational.

$0.1^2 = 0.01$

$0.2^2 = 0.04$

$0.3^2 = 0.09$

$0.4^2 = 0.16$

$0.5^2 = 0.25$

$0.6^2 = 0.36$

$0.7^2 = 0.49$

$0.8^2 = 0.64$

$0.9^2 = 0.81$